Lecture 4-1

Canonical And Standard Forms

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Key Terms

- **Literal - a variable or the complement of a variable.**
  ex) X, Y, $\overline{X}$

- **Normal Term - a product or sum term in which no var. appears more than once.**
  ex) XYZ, Y+W

- **Product Term (AND term) : a single literal or a product of two or more literals.**
  ex) Z, XY, $W\overline{Y}Z$, $\overline{W}\cdot\overline{Y}\cdot Z$.
  A product term can be represented by a rectangle (cube) in a K-map and we will see why later.

- **Sum Term (OR term) ?**

Minterms

- Now consider two binary variables X and Y combined with an AND operation.
- There are four possible combinations: $X' \cdot Y'$, $X' \cdot Y$, $X \cdot Y'$, and $X \cdot Y$.
- Each of these four AND terms is called a minterm.

X	Y	Terms	Designation
0	0	$X' \cdot Y'$	m_0
0	1	$X' \cdot Y$	m_1
1	0	$X \cdot Y'$	m_2
1	1	$X \cdot Y$	m_3

Maxterms

- Now consider two binary variables X and Y combined with an OR operation.
- There are four possible combinations: X'+Y', X'+Y, X+Y', and X+Y.
- Each of these four OR terms is called a maxterm.

X	Y	Terms	Designation
0	0	X+Y	M_0
0	1	X+Y'	M_1
1	0	X'+Y	M_2
1	1	X'+Y'	M_3

Sum of Minterms

- A Boolean function can be expressed algebraically from a given truth table by forming a minterm for each combination of the variables that produces a 1 in the function and then taking the OR of all those terms.

- Any Boolean function can be expressed as a sum of minterms.

- Let's derive the Boolean function from a truth table.

X	Y	F	Designation
0	0	1	m_0
0	1	0	m_1
1	0	1	m_2
1	1	0	m_3

$F = X'Y' + XY' = m_0 + m_2$

- $F(X, Y) = \Sigma(0, 2) = m_0 + m_2$ [brief notation]

Product of Maxterms

- Any Boolean function can be expressed as a product of maxterms.

- Form a maxterm for each combination of the variables that produces a 0 in the function, and then from the AND of all those maxterms.

- In the former example,

$$F = (X+Y')(X'+Y') = M_1 \cdot M_3$$

$$F(X, Y) = \Pi\,(1, 3) \quad \text{[brief notation]}$$

- Boolean functions expressed as a sum of minterms or product of maxterms are said to be in canonical form.

Example : Function of 3 Variables

- **Let's derive the logical expression as a sum of minterms from a truth table.**

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$$F = X'Y'Z + XY'Z' + XYZ$$
$$= m_1 + m_4 + m_7 = \Sigma(1, 4, 7)$$

- **As a product of maxterms,**

$$F = \Pi(0, 2, 3, 5, 6) = M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6$$

Conversion between Canonical Forms

- **As an Example, $F(X,Y,Z) = \Sigma\,(1, 4, 7)$**

  - the complement of F, $F'(X,Y,Z) = \Sigma\,(0, 2, 3, 5, 6)$
  $= m_0 + m_2 + m_3 + m_5 + m_6$

  - the complement of F' by DeMorgan's theorem

  $$F = (m_0 + m_2 + m_3 + m_5 + m_6)' = m_0' \cdot m_2' \cdot m_3' \cdot m_5' \cdot m_6'$$
  $$= M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6$$
  $$= \Pi\,(0, 2, 3, 5, 6)$$

  - $m_j' = M_j$ **; the maxterm with subscript j is a complement of the minterm with the same subscript j and vice versa.**
  ex) $m_0' = (X' \cdot Y' \cdot Z')' = X + Y + Z = M_0$

- **To convert from one canonical form to another, interchange the symbols Σ and Π and list those numbers missing from the original form.**

Standard Forms

- **The two canonical forms of Boolean algebra are basic forms that one obtains from reading a given function from the truth table.**

- **Another way to express Boolean functions is in standard form. There are two types of standard forms : the sum of products and products of sums.**

- **Sum-of-product (=SOP) : a sum of product terms.**

  ex) $\overline{Y} + XY + \overline{X}Y\overline{Z}, X\overline{Y}+Y+Z$

- **Product-of-Sums (POS) ?**

- **This standard type of expression results in a two-level implementa-tion.**