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Lecture 4-1

Canonical And Standard Forms

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Key Terms

- Literal a variable or the complement of a variable.
 ex) X, Y, X
- Normal Term a product or sum term in which no var. appears more than once.

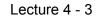
ex) XYZ, Y+W

 Product Term (AND term) : a single literal or a product of two or more literals.

ex) Z, XY, WYZ, W·Y·Z.

A product term can be represented by a rectangle (cube) in a Kmap and we will see why later.

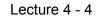
• Sum Term (OR term) ?



Minterms

- Now consider two binary variables X and Y combined with an AND operation.
- There are four possible combinations: X'.Y', X'Y, X.Y', and X.Y.
- Each of these four AND terms is called a minterm.

Χ	Y	Terms	Designation
0	0	X'.Y'	m₀
0	1	Χ' · Υ	m₁
1	0	X·Y'	m ₂
1	1	X·Y	m₃



Maxterms

- Now consider two binary variables X and Y combined with an OR operation.
- There are four possible combinations: X'+Y', X'+Y, X+Y', and X+Y.
- Each of these four OR terms is called a maxterm.

Χ	Y	Terms Designation	
0	0	X+Y	Μο
0	1	X + Y '	M 1
1	0	X' + Y	M 2
1	1	X' + Y'	M ₃



Sum of Minterms

- A Boolean function can be expressed algebraically from a given truth table by forming a minterm for each combination of the variables that produces a 1 in the function and then taking the OR of all those terms.
- Any Boolean function can be expressed as a sum of minterms.
- Let's derive the Boolean function from a truth table.

Χ	Υ	F	Designation	
0	0	1	m₀	
0	1	0	m 1	
1	0	1	m ₂	
1	0 1 0 1	0	m 3	$F = X'Y' + XY' = m_0 + m_2$
• F(X,Y) = Σ (0, 2) =), 2) = m₀+ n	n2 [brief notation]

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Product of Maxterms

- Any Boolean function can be expressed as a product of maxterms.
- Form a maxterm for each combination of the variables that produces a 0 in the function, and then from the AND of all those maxterms.
- In the former example,

 $F = (X+Y')(X'+Y') = M_1 \cdot M_3$

 $F(X, Y) = \Pi(1, 3)$ [brief notation]

• Boolean functions expressed as a sum of minterms or product of maxterms are said to be in canonical form.

Example : Function of 3 Variables

• Let's derive the logical expression as a sum of minterms from a truth table.

Χ	Y	Ζ	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

 $\mathbf{F} = \mathbf{X}'\mathbf{Y}'\mathbf{Z} + \mathbf{X}\mathbf{Y}'\mathbf{Z}' + \mathbf{X}\mathbf{Y}\mathbf{Z}$

 $= m_1 + m_4 + m_7 = \Sigma (1, 4, 7)$

• As a product of maxterms,

 $F = \Pi (0, 2, 3, 5, 6) = M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6$

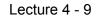
Conversion between Canonical Forms

- As an Example, $F(X,Y,Z) = \Sigma$ (1, 4, 7)
 - the complement of F, $F'(X,Y,Z) = \Sigma (0, 2, 3, 5, 6)$ = $m_0 + m_2 + m_3 + m_5 + m_6$
 - the complement of F' by DeMorgan's theorem

$$F = (m_0 + m_2 + m_3 + m_5 + m_6)' = m_0' \cdot m_2' \cdot m_3' \cdot m_5' \cdot m_6'$$

= $M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6$
= $\Pi (0, 2, 3, 5, 6)$

- mj' = Mj ; the maxterm with subscript j is a complement of the minterm with the same subscript j and vice versa.
 mo' = (X'·Y'·Z')' = X + Y + Z = Mo
- To convert from one canonical form to another, interchange the symbols Σ and Π and list those numbers missing from the original form.



Standard Forms

- The two canonical forms of Boolean algebra are basic forms that one obtains from reading a given function from the truth table.
- Another way to express Boolean functions is in standard form. There are two types of standard forms : the sum of products and products of sums.
- **Sum-of-product** (=**SOP**) : a sum of product terms.

ex) \overline{Y} + XY + $\overline{X}Y\overline{Z}$, X \overline{Y} + Y + Z

- Product-of-Sums (POS) ?
- This standard type of expression results in a two-level implementation.