Lecture 4

Switching Algebra

Revised by WJ Han

Youpyo Hong, Dongguk University

Postulates & Theorems 1

- **Postulate 2** $X + 0 = X$, $X \cdot 1 = X$
- **Postulate 5** $X + \overline{X} = 1$, $X \cdot \overline{X} = 0$
- **Theorem 1** $X + X = X$, $X \cdot X = X$
- **Theorem 2** $X + 1 = 1$, $X \cdot 0 = 0$

Postulates & Theorems 2

- **Commutative** $X + Y = Y + X$, $X \cdot Y = Y \cdot X$
- **Associative** $(X + Y) + Z = X + (Y + Z)$, $(X \cdot Y) \cdot Z = X \cdot (Y \cdot Z)$
- **Distributive** $X \cdot Y + X \cdot Z = X \cdot (Y + Z)$, $(X+Y)(X+Z)=X + Y \cdot Z$
- **DeMorgan** $\overline{(X + Y)} = \overline{X} \cdot \overline{Y}$, $\overline{(XY)} = \overline{X} + \overline{Y}$
- **Absorption** $X + X \cdot Y = X$, $X \cdot (X+Y) = X$

Algebraic Manipulation

$$F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$$

$$= \overline{X}Y(Z + \overline{Z}) + XZ$$

$$= \overline{X}Y \cdot 1 + XZ$$

$$= \overline{X}Y + XZ$$

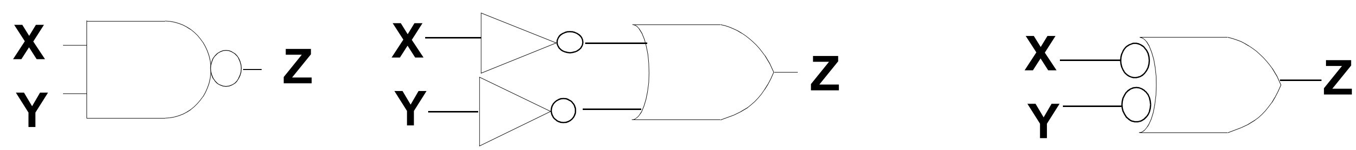
DeMorgan's Law

$$\overline{X + Y} = \overline{X} \cdot \overline{Y} \qquad \overline{X \cdot Y} = \overline{X} + \overline{Y}$$

=> Gate Conversion

Gate Conversion

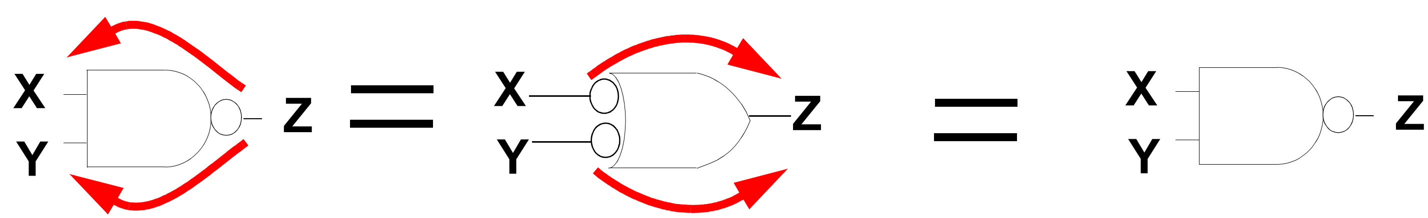
If $Z = \overline{XY}$, then $Z = \overline{X} + \overline{Y}$



If $Z = \overline{X+Y}$, then $Z = \overline{X} \cdot \overline{Y}$

?

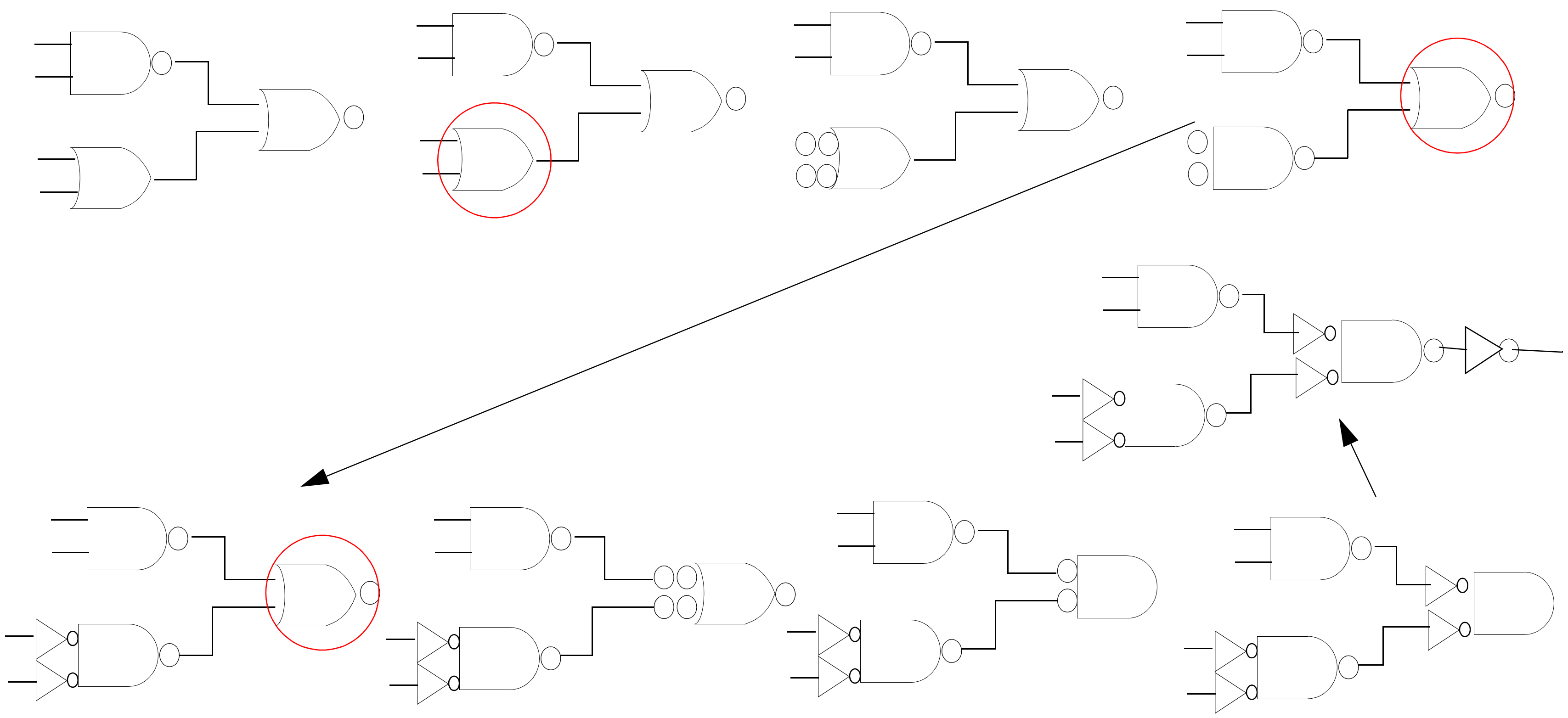
Note : You can change OR to AND by shifting bubbles across the gate.



If there are more than one gate, convert from input to output.

Multiple Gates Conversion Example

- **Redraw the first logic diagram using NAND gates and inverters only.**



Finding a Complement

Change OR to AND, and AND to OR, then complement each variable.

$$F = \bar{X}Y\bar{Z} + \overline{XY}Z$$

Find $\bar{F}$.

$$\bar{F} = \overline{\bar{X}Y\bar{Z} + \overline{XY}Z} =$$

Duality

- **Principle of Duality**

  - Any expression remains true if 0 and 1 are swapped and AND and OR are swapped from any Boolean theorem.

  Ex) Dual of {X + Y = Y + X} is {XY = YX}

  - Before taking dual, fully parenthesize the formula.

  Ex) Find the dual of {X + X·Y = X}

  $X \cdot X + Y = X$

  $X + Y = X$ **(Wrong)**

  $X + (X \cdot Y) = X$

  $X \cdot (X + Y) = X$

  $X + X \cdot Y = X$

Proof Practice 1

Prove $A(A+B) = A$

$A \cdot (A+B)$

$= A \cdot A + AB$

$= A + A \cdot B$

$= A \cdot 1 + A \cdot B$

$= A \cdot (1+B)$

$= A \cdot 1$

$= A$

Proof Practice 2

Prove $A + \overline{A} \cdot B = A + B$

$A + \overline{A} \cdot B = A \cdot 1 + \overline{A} \cdot B$

$= A \cdot (B + \overline{B}) + \overline{A} \cdot B$

$= A \cdot B + A \cdot \overline{B} + \overline{A} \cdot B$

$= A \cdot B + A \cdot B + A \cdot \overline{B} + \overline{A} \cdot B$

$= A \cdot B + A \cdot \overline{B} + A \cdot B + \overline{A} \cdot B$

$= A \cdot (B + \overline{B}) + B \cdot (A + \overline{A})$

$= A \cdot 1 + B \cdot 1$

$= A + B$

Proof Practice 3

Proof of $A + \overline{A} \cdot B = A + B$ using its dual.

The dual of the given formula = $A \cdot (\overline{A} + B) = A \cdot B$

$A \cdot (\overline{A} + B)$

$= A \cdot \overline{A} + A \cdot B$

$= 0 + A \cdot B$

$= A \cdot B$

Thus, $A + \overline{A} \cdot B = A + B$