# Lecture 5

# Karnaugh-Map

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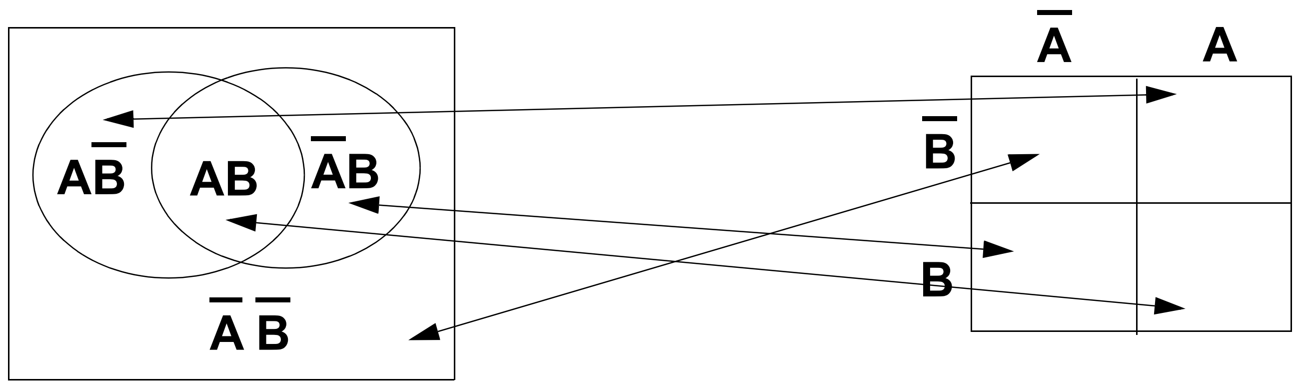
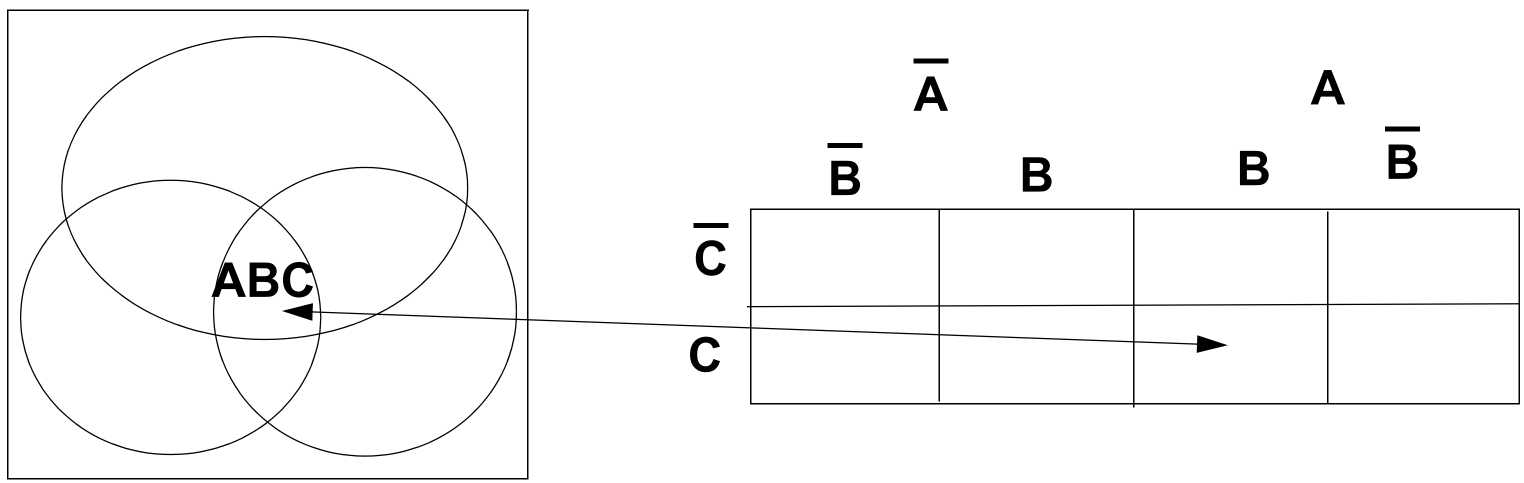
# Karnaugh-Map

# K-Map for 2 Variables



# K-Map for 3 Variables



# Logic Expression, Truth Table, K-Map, and Logic Diagram

$$F = \overline{A}\,\overline{B} + AB$$

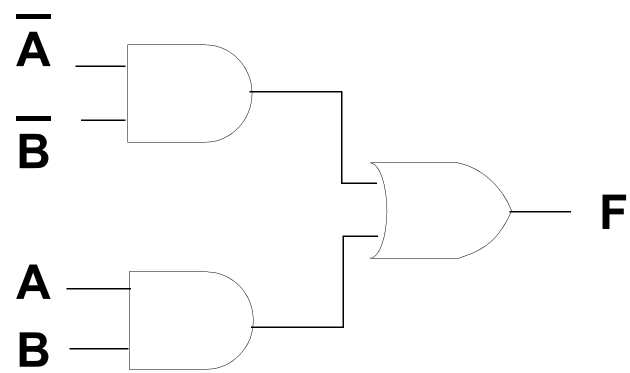
Logic Expression

| | $\overline{A}$ | $A$ |
|---|---|---|
| $\overline{B}$ | 1 | 0 |
| $B$ | 0 | 1 |

K-Map

| A | B | F |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

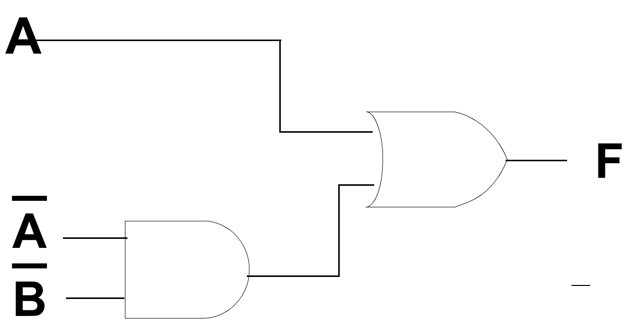
**Truth Table**




Logic Diagram
(=Digital Circuit Representation)

# Non-Unique Circuits for the Same Function

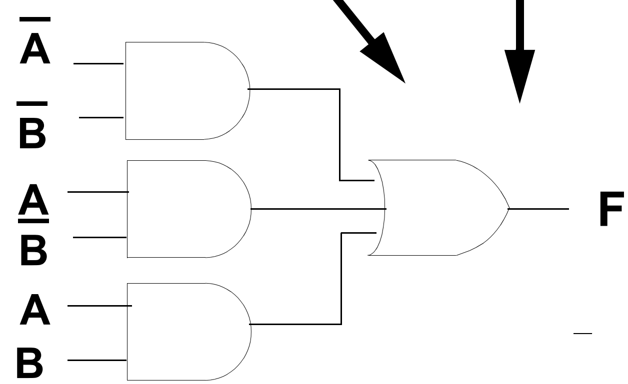
$F = A + \overline{A} \cdot \overline{B}$




Because $F = A + \overline{A} \cdot \overline{B}$

$= A \cdot (B + \overline{B}) + \overline{A} \cdot \overline{B} = AB + A \cdot \overline{B} + \overline{A} \cdot \overline{B}$

| | $\overline{A}$ | $A$ |
|---|---|---|
| $\overline{B}$ | 1 | 1 |
| $B$ | 0 | 1 |



# Truth Table Again

- Let's derive the logical expression from a truth table.

| X | Y | Z | F |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

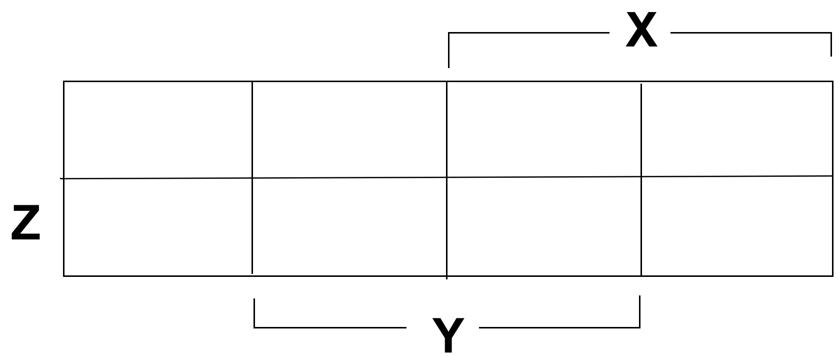
$$F = X \cdot \overline{Y} \cdot \overline{Z} + X \cdot Y \cdot \overline{Z}$$

$$(= X\overline{Z})$$

- A truth table provides a Sum-of-Minterms form naturally.

- A K-map also provides a Sum-of-Product (SOP) form naturally.
  : each product does not have to be a minterm as we will see.

# K-Map Indexing Methods




| Z \ XY | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | | | | |
| 1 | | | | |

Our Choice

# Property of K-Map

- **One cell in K-map represents a minterm.**
- **We can get an AND-OR (Sum-of-Product) style circuits "easily" from K-map.**
- **Two adjacent cells in K-map contain the same variable in positive and negative forms.**

| Z \ XY | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| **0** | 0 | 0 | **1** | **1** |
| **1** | 0 | 0 | 0 | 0 |

$X \cdot \overline{Y} \cdot \overline{Z}$

$X \cdot Y \cdot \overline{Z}$

# Adjacency in K-Map (1)

**A cell is adjacent to the cells if they share a line..**

| Z \ XY | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 |

| YZ \ WX | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | 1 | 1 |
| 01 | 0 | 1 | 1 | 1 |
| 11 | 0 | 0 | 1 | 0 |
| 10 | 0 | 0 | 1 | 1 |

# Adjacency in K-Map (2)

**Note that the cell on the boundary are adjacent to the cell on the other side. (You need some imagination power!!)**

| Z \ XY | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |

| YZ \ WX | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | 1 | 1 |
| 01 | 0 | 0 | 1 | 0 |
| 11 | 0 | 0 | 0 | 0 |
| 10 | 0 | 1 | 1 | 1 |

# Motivation of K-Map Simplification

- If we express K-map using logic expression, only the cells with 1 show up in Sum-of-Minterms form.

| Z \ XY | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |

$X\cdot\overline{Y}\cdot\overline{Z}$

$X\cdot Y\cdot\overline{Z}$

$$F = X\cdot\overline{Y}\cdot\overline{Z} + X\cdot Y\cdot\overline{Z}$$

- Two minterms that differ only in a variable can be combined into one.

Ex) $X\cdot\overline{Y}\cdot\overline{Z} + X\cdot Y\cdot\overline{Z} = X\overline{Z}(Y+\overline{Y}) = X\overline{Z}$

- We like simpler form. How can we get simpler expression from K-map?

# Introduction to K-Map Simplification

- We use a rectangle to specify that the sum of two minterms in that rectangle can be combined to only one product term.

| Z \ XY | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| **0** | 0 | 0 | 1 | 1 |
| **1** | 0 | 0 | 0 | 0 |

$X\cdot\overline{Z}$

- How to read the expression for a rectangle?

  - Find out the variable whose polarity (positive or negative) is consistent in the rectangle.

# Exercise on a Rectangle Reading

| Z \ XY | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 |

| YZ \ WX | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 1 | 1 |
| 01 | 0 | 1 | 1 | 0 |
| 11 | 0 | 1 | 1 | 0 |
| 10 | 0 | 0 | 1 | 1 |

# Minterm Numbers in Truth Table and K-Map

| X | Y | Z | minterm no. |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 2 |
| 0 | 1 | 1 | 3 |
| 1 | 0 | 0 | 4 |
| 1 | 0 | 1 | 5 |
| 1 | 1 | 0 | 6 |
| 1 | 1 | 1 | 7 |

| Z \ XY | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | *0* | *2* | *6* | *4* |
| 1 | *1* | *3* | *7* | *5* |

**$F = \overline{X}\cdot\overline{Y}\cdot\overline{Z} + X\cdot Y\cdot Z$ can be described by $F = \sum (0, 7)$**

# K-Map Simplification Examples

$F = \sum (0,1,2,3)$

| A | B | C | F |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

| C \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 |

$F = \bar{A}\,\bar{B}\,\bar{C} + \bar{A}\,\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC = \bar{A}\,\bar{C} + \bar{A}C$

| C \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 |

$F = \bar{A}\,\bar{C} + \bar{A}C = \bar{A}$

| Z \ XY | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |

| Z \ XY | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |

| Z \ XY | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |

$$F = \overline{X}\cdot\overline{Z} + X\cdot Y\cdot\overline{Z}$$

| Z \ XY | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |

$$F = \overline{X}\cdot\overline{Z} + Y\cdot\overline{Z} \text{ (Better)}$$

- **Lesson**

  - **Combine cells using as large rectangle as possible.**

| Z \ XY | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | | |
| 1 | | 1 | | |

**Impossible cover**

**(Only cells in a rectangle can be circled. Why?)**

| Z \ XY | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | | |
| 1 | | 1 | 1 | |

**Redundant cover**

- **Lesson**

  **- Use as few rectangles as possible.**

# Four-Variable K-Map

| CD \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 4 | 12 | 8 |
| 01 | 1 | 5 | 13 | 9 |
| 11 | 3 | 7 | 14 | 11 |
| 10 | 2 | 6 | 15 | 10 |

$A \cdot D$

**Minterm Numbers in K-Map**

**$F = A \cdot D$ can be described by $F = \Sigma\,(9,11,13,14)$**

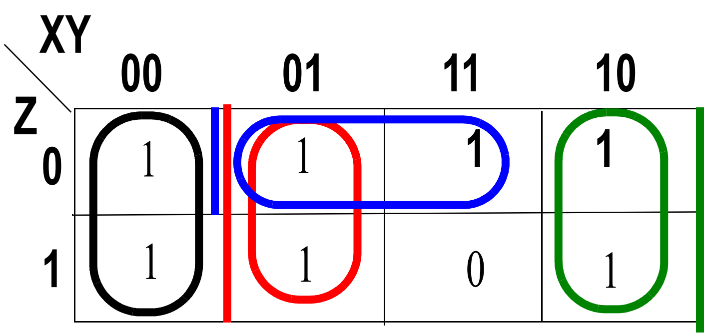
# The Covers That Can Be Combined

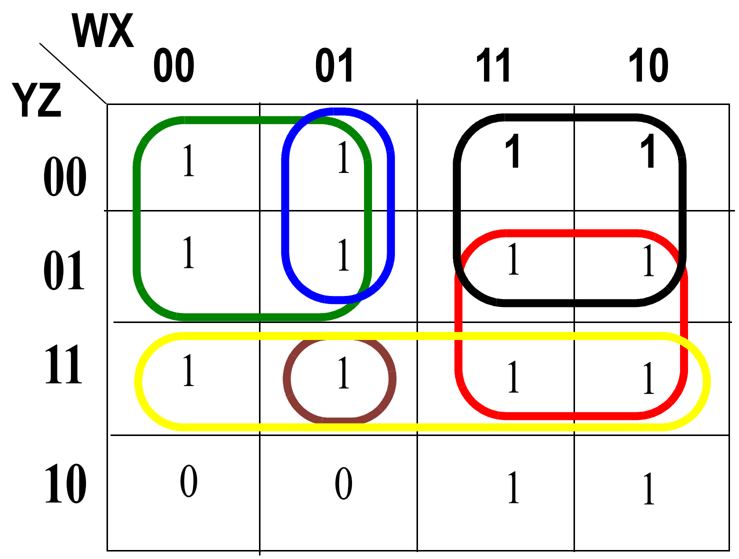
- They must be adjacent.
- Their sizes are the same.
- Correct covers can have $2^n$ cells in it.

$2 + 2 = 4$, $4 + 4 = 8$, $8 + 8 = 16$, so on.....

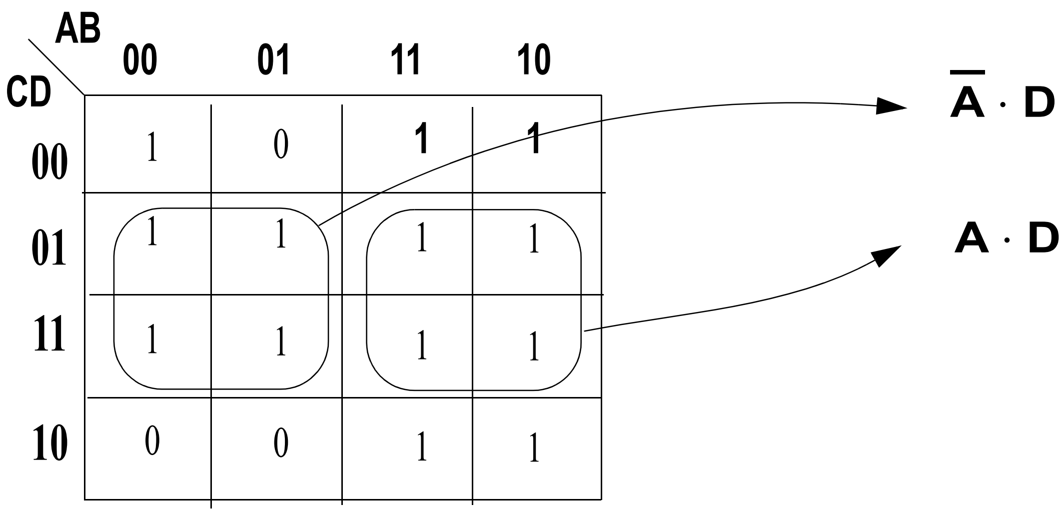
# Adjacency in K-Map (3)

**A cube is adjacent to another cube if they share a segment along one side and their sizes are the same.**



# Adjacent Covers

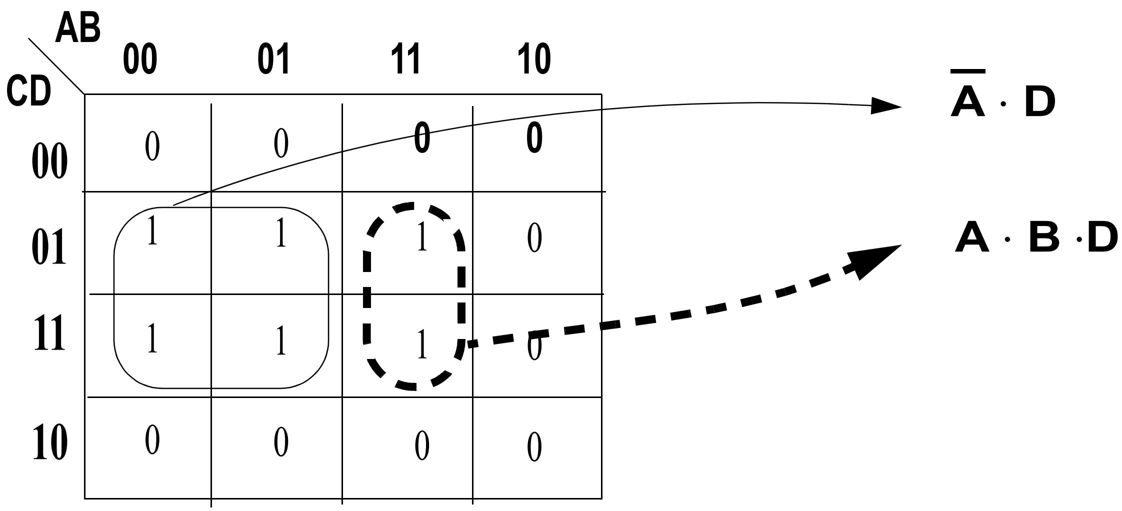



**The two covers differ (complement) in only one variable**

**if the two covers are adjacent.**

**Then the two covers can be combined into one cover.**

# Adjacent Covers with Different Sizes

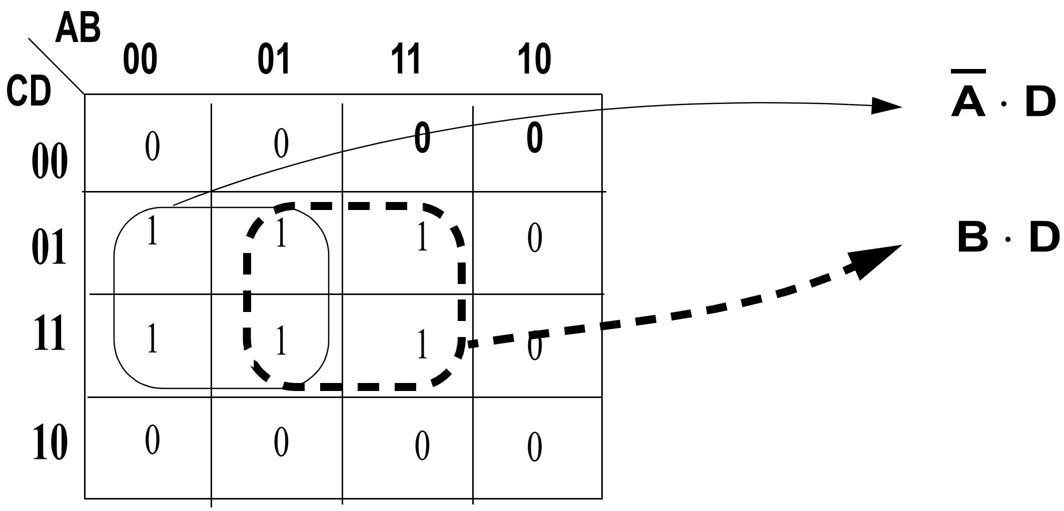



**The two covers have different size.**

**The two covers differ in more than one variables.**

**Then the two covers cannot be combined.**

# Partially Overlapped Covers




**Combine covers using as large cover as possible.**

**$F = \overline{A} \cdot D + B \cdot D$ is better than $F = \overline{A} \cdot D + A \cdot B \cdot D$**

# Exercise on Covers

| Z \ XY | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

| Z \ XY | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 |

| CD \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 1 | 0 |
| 01 | 1 | 1 | 1 | 0 |
| 11 | 1 | 1 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 |

| CD \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | 1 | 0 |
| 01 | 1 | 1 | 1 | 0 |
| 11 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 |

# Practice

- Simplify the Boolean function

$$F(W,X,Y,Z) = \Sigma\,(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$

- Simplify the Boolean function

$$F = \overline{A}\cdot\overline{B}\cdot\overline{C} + \overline{B}\cdot C\cdot\overline{D} + \overline{A}\cdot B\cdot C\cdot\overline{D} + A\cdot\overline{B}\cdot\overline{C}$$